Vectors

Basic Properties of Vectors

- If B is a point with the coordinates (x, y), then $\overrightarrow{OB}$ can be expressed as a column vector $\binom{x}{y}$ as O represents the origin and has the coordinates $(0,0)$.
  The magnitude/length of $\overrightarrow{OB}$ is thus $|\overrightarrow{OB}| = \sqrt{x^2 + y^2}$
- Reversing the direction of $\overrightarrow{OB}$ thus gives us $\overrightarrow{BO}$ and is denoted by $\binom{-x}{-y}$
- A column vector can be interpreted as the movement from one point to another. For $\overrightarrow{OB} = \binom{h}{k}$, it means that from point O to point B the x −coordinate shifted by h units and the y −coordinate shifted by k units.

Background knowledge for Vectors

- $\binom{a}{b} + \binom{c}{d} = \binom{a+c}{b+d}$
- $\binom{a}{b} - \binom{c}{d} = \binom{a-c}{b-d}$
- $k\binom{a}{b} = \binom{ka}{kb}$

If u, v and w are vectors, and m and n are constants, then

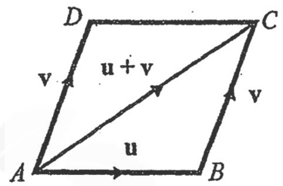
- $u + v = v + u$
- $(u + v) + w = u + (v + w)$
- $u - v = u + (-v)$
- $m(nu) = n(mu) = (nm)u$
- $(m + n)u = mu + nu$
- $m(u + v) = mu + mv$

Position Vectors

- Any vector taken with respect to the origin is known as a position vector (eg $\overrightarrow{OB}, \overrightarrow{OA}$).
- Vectors can be expressed as subtraction of position vectors:

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

Addition of Vectors



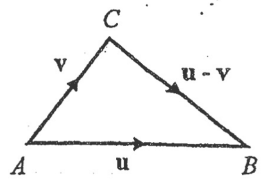
Triangle Law of Addition

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

Parallelogram Law of Addition

$$\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$$

Subtraction of Vectors



If $\overrightarrow{AC} = v$, then $\overrightarrow{CA} = -v$ and $u - v = u + (-v)$

Applications of Vectors in Geometry

- If $\overrightarrow{AB} = \overrightarrow{CD}$ then AB and CD are parallel and $AB = CD$
- If $\overrightarrow{AB} = k\overrightarrow{CD}$ then AB and CD are parallel and they are in the same direction if k is positive. Otherwise, they will be in the opposite direction

Describing Vectors

When asked to describe vectors, there are 3 things we can mention, namely:

1. Direction ($\overrightarrow{AB} = 2\overrightarrow{CD}$, hence they face the same direction)
2. Magnitude ($\overrightarrow{AB} = 2\overrightarrow{CD}$, hence AB is twice the length of CD)
3. Collinearity ($\overrightarrow{AB} = 2\overrightarrow{BD}$, hence points A, B and D lie on the same line)

Ratio of Areas

2 main ways that area questions can be asked (or as a combination of both):

1. Common height (h)

$$\frac{A_1}{A_2} = \frac{\frac{1}{2} \times B_1 \times h}{\frac{1}{2} \times B_2 \times h} = \frac{B_1}{B_2}$$

Ratio of their areas is a ratio of their bases.

2. Similar Figures

If it is proven that the figures are similar, then

$$\frac{A_1}{A_2} = \left(\frac{S_1}{S_2}\right)^2$$

Ratio of their areas is a ratio of their sides squared.

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